PEARSON CLINICAL LABORATORY SCIENCE SERIES

# Clinical Laboratory Mathematics

Mark D. Ball

ELIZABETH A. GOCKEL-BLESSING, SERIES EDITOR

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# Contents

# Foreword

*Clinical Laboratory Mathematics* is part of Pearson's Clinical Laboratory Science series of textbooks, which is designed to balance theory and practical applications in a way that is engaging and useful to students. The author of *Clinical Laboratory Mathematics* presents highly detailed technical information and effective tools that will help beginning learners envision themselves as members of a healthcare team, while helping advanced learners and practitioners continue their education. The synergy of theoretical and practical information in this text enables learners to analyze data and synthesize conclusions. Additional applications and instructional resources are available at *www.myhealthprofessionskit.com*.

We hope that this book, as well as the entire series, proves to be a valuable educational resource.

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# Preface

*Clinical Laboratory Mathematics* is a comprehensive textbook on the mathematical techniques and theories of clinical laboratory science. It is written for students at any point on the trajectory toward an undergraduate or graduate degree in the discipline, from an associate's degree to a doctorate. Students and practitioners of related disciplines will also find the book useful: pathologists, medical students, nurses, pharmacists, biochemists, biomedical engineers, and physician assistants.

Going well beyond the notion of "relevance," this book tries to convey the conviction that learning mathematics is not only helpful, but often critical, in the high-technology milieu of a clinical laboratory. It repeatedly highlights the reasons for developing a battery of mathematical tools: (1) to handle unfamiliar mathematical problems that arise in the course of laboratory work; (2) to follow the reasoning in seminars, papers, and discussions; (3) to detect mathematical errors made by individuals; (4) to recognize instrument malfunctions or method anomalies through mathematical irregularities; (5) to adapt new methods, ideas, and technologies that require some mathematical competence; and (6) to shift smoothly into research-oriented work, whether in the form of short-term projects in a routine laboratory, long-term projects in a research laboratory, or method development at a diagnostics company.

Therefore, the book integrates real-world examples of mathematical tools at work in the clinical laboratory. To achieve this goal, practice problems are strategically designed to have the student confront scenarios involving mathematical questions that have both context and consequence. Such problems offer the student a chance to think under the circumstances that a laboratory professional might encounter on the job, requiring him or her to solve a mathematical problem while coming to appreciate the importance of correct calculation and the repercussions of error.

The book supports both self-guided study and the more traditional lecture-discussion format. Meeting the needs of either approach, or of any approach in-between, is a matter not only of organizing the topics logically, but also of liberally cross-referencing so that students see connections and common motifs. This technique promotes comprehension while lessening the burden of brute memorization.

The book includes online resources (*www.myhealthprofessionskit.com*) intended to meet the needs of advanced users: (1) chapter appendices, which elaborate topics introduced in the main text, and (2) advanced topics, which emerge from frequently asked questions and from the main text.

Because some instructors start their courses with a review of arithmetic, and because some students seek such a review, the first chapter deals with addition, subtraction, multiplication, division, fractions, decimals, percentages, algebra, and ratios. Furthermore, it includes strategies for speeding up calculations without relying on electronics. Subsequent chapters cover increasingly complex and specialized topics, with the online appendices carrying those topics to the greatest depth.

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# 1 Arithmetic and Algebra

## Learning Objectives

At the end of this chapter, the student should be able to do the following:

1. To add, subtract, multiply, and divide positive and negative numbers
2. To multiply, divide, and reduce fractions
3. To add and subtract fractions
4. To express fractions as decimal numbers and to express improper fractions as mixed numbers
5. To simplify complex fractions
6. To interconvert percentages, decimal numbers, and fractions
7. To calculate a specified percentage of a number
8. To express change properly as a percentage
9. To solve an equation algebraically for an unknown variable
10. To calculate and interpret ratios
11. To solve equations of two ratios for an unknown variable by cross-multiplication

## Key Terms

associative property
canceling
commutative property
complex fraction
denominator
difference
distributive property
factor
improper fraction
least common denominator
mixed number
numerator
opposite
percentage
product
proper fraction
quotient
ratio
reciprocal
reducing
sum

## Chapter Outline

Arithmetic is the manipulation of numbers through addition, subtraction, multiplication, and division. Algebra is the strategic manipulation of relationships in order to find the unknown value of a certain quantity. In medical decisions, the importance of having reliable information is self-evident. Therefore, mastering the basic skills of arithmetic and algebra is critical to ensuring the accuracy of every result that leaves the laboratory.

## ADDITION

In the problem

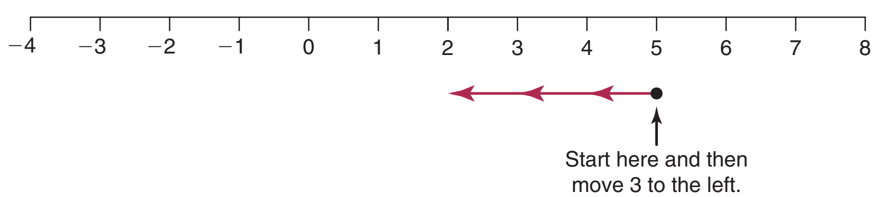
$$a + b = c$$

variable *c* is referred to as the **sum** of *a* and *b*.

In the operation of addition, positive numbers represent a "putting in" and negative numbers a "taking out." Therefore, we regard a positive number and its negative counterpart as **opposites**. For example, the opposite of "7" is "−7," and the opposite of "−200" is "200." Consequently, combining a positive number with a negative number amounts to a decrease. For example,

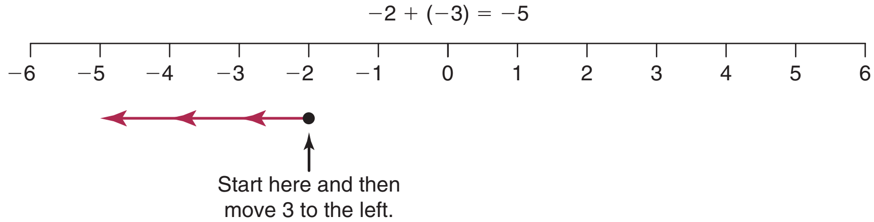
$$5 + (-3) = 2$$

A simple way to approach a problem like this is to refer to a number line. Adding a negative number is the same as moving leftward. In this case, we start at the "5" and then move to the left by "3," which brings us to "2."




Adding a negative number to a negative number follows the same rule, that is, a leftward movement:

$$-2 + (-3) = -5$$




To clarify this procedure with an analogy, envision a beaker of water on a tabletop. Let the number "1" be a unit of heat and the number "−1" be a unit of cold. Adding a positive number to another positive number puts units of heat into the water, causing the temperature to rise. Adding a negative number to a positive number, however, introduces units of cold to the water, bringing the temperature down.

Addition is commutative. In other words, the order in which we add two numbers together does not affect the sum. Thus, this equation shows the **commutative property** of addition, that is, adding *a* and *b* gives the same result as adding *b* and *a*:

$$a + b = b + a$$

For example,

$$3 + 6 = 6 + 3 = 9$$

and

$$-0.721 + 0.0044 = 0.0044 + (-0.721)$$

The grouping of numbers in addition also does not affect the sum. This fact reflects the **associative property** of addition, meaning the sum of *a* and *b* plus *c* is equal to *a* plus the sum of *b* and *c*, as represented in this equation:

$$(a + b) + c = a + (b + c)$$

For example,

$$(2 + 8) + 5 = 2 + (8 + 5) = 15$$

and

$$(-1 + 9) + 3 = -1 + (9 + 3) = 11$$

### ☑ CHECKPOINT 1-1

1. Evaluate the following expressions.
   (a) $16 + (-9)$ (b) $(-4) + 10$ (c) $1.7 + (-3.4)$ (d) $(-58) + (-4)$
2. Evaluate the following expressions.
   (a) $(-9) + 5 + (-2)$ (b) $13.5 + 0.2 + (-0.8)$
   (c) $0.0556 + (-0.0102) + 0.0433$ (d) $(-128) + (-128) + 256$

1. (a) 7 (b) 6 (c) −1.7 (d) −62
2. (a) −6 (b) 12.9 (c) 0.0887 (d) 0

## SUBTRACTION

In the problem

$$a - b = c$$

variable *c* is referred to as the **difference** between *a* and *b*.

Subtracting a positive number from a positive number is intuitive:

$$13 - 8 = 5$$

In fact, we define subtraction as the addition of an opposite:

$$a - b = a + (-b)$$

Subtracting a negative number from a positive number, however, may seem counterintuitive:

$$13 - (-8) = 21$$

Here, we are subtracting the opposite of 8 from 13. If we were subtracting 8 itself, then we would bring the total down to 5, that is, $13 - 8 = 5$. Instead, we are subtracting a "taking out," a process that amounts to a "putting in." Therefore, subtracting a negative number has the same effect as adding its opposite:

$$13 - (-8) = 13 + 8 = 21$$

Our beaker-of-water analogy might prove helpful here. We can say that subtracting a negative is the same as withdrawing units of cold from the water, the result of which is an *increase* in the temperature.

### ☑ CHECKPOINT 1-2

Evaluate the following expressions.
(a) $10 - (-2)$ (b) $(-3) - 5$ (c) $40 - 46$ (d) $(-18) - (-30)$

(a) 12 (b) −8 (c) −6 (d) 12

## MULTIPLICATION

In the problem

$$a \times b = c$$

variables *a* and *b* are called the **factors**, and variable *c* is referred to as the **product** of *a* and *b*.

Multiplication is a shortcut for addition:

$$6 \times 4 = 24$$

What this operation does is to add together six fours or four sixes:

$$6 \times 4 = 4 + 4 + 4 + 4 + 4 + 4 = 6 + 6 + 6 + 6 = 24$$

There are three common ways to symbolize multiplication:

$$a \times b = a \cdot b = ab$$

Like addition, multiplication is commutative. The order in which we multiply two numbers together does not affect the product:

$$a \times b = b \times a$$

For example,

$$6 \times 5 = 5 \times 6 = 30$$

The grouping of numbers in multiplication also does not affect the product. Thus, the associative property of multiplication is

$$(a \times b) \times c = a \times (b \times c)$$

For example,

$$(3 \times 7) \times 2 = 3 \times (7 \times 2) = 42$$

As in addition and subtraction, multiplying two positive numbers together makes sense. Equally logical, though, is multiplying a positive number by a negative number:

$$6 \times (-4) = -24$$

What this operation does is to add together six negative fours or negative-six fours:

$$6 \times (-4) = (-4) + (-4) + (-4) + (-4) + (-4) + (-4) = -24$$

$$(-6) \times 4 = -24$$

What does it mean to add together negative-six fours? Fortunately, our beaker-of-water analogy is useful here, too. Regard the operation not as an *addition* of negative-six fours but as a *subtraction* of six fours, giving −24. In other words, we are subtracting four units of heat six times, for a total of 24 units of heat out of the water. The result is a *lower* temperature. Therefore, a negative times a positive is a negative.

Another way to approach this problem is to apply the commutative property of multiplication:

$$(-6) \times 4 = 4 \times (-6) = -24$$

Written as such, the problem tells us simply to add together four negative sixes:

$$4 \times (-6) = (-6) + (-6) + (-6) + (-6) = -24$$

Finally, consider the multiplication of two negative numbers:

$$(-6) \times (-4) = 24$$

To understand this, we can extend our analogy from above and treat the operation as a subtraction of six negative fours, giving 24. In other words, we are subtracting, or withdrawing, four units of cold six times, pushing the temperature *up*. Therefore, a negative times a negative is a positive.

Table 1-1 ★ summarizes the four possible sign combinations in multiplication.

★ **TABLE 1-1** The Four Sign Combinations in Multiplication

| Rule | Analogy |
|---|---|
| positive × positive = positive | Adding units of heat raises the temperature |
| positive × negative = negative | Adding units of cold lowers the temperature |
| negative × positive = negative | Subtracting units of heat lowers the temperature |
| negative × negative = positive | Subtracting units of cold raises the temperature |

## ☑ CHECKPOINT 1-3

Evaluate the following expressions.

| | | | |
|---|---|---|---|
| (a) $4 \times 9$ | (b) $2 \times (-6)$ | (c) $(-10) \times 3$ | (d) $(-5) \times (-4)$ |
| (e) $1.5(2)$ | (f) $33 \cdot (-3)$ | (g) $(-8)(-8)$ | (h) $(-4.04) \cdot 2$ |

| | | | |
|---|---|---|---|
| (a) 36 | (b) −12 | (c) −30 | (d) 20 |
| (e) 3 | (f) −99 | (g) 64 | (h) −8.08 |

## DIVISION

In the problem

$$a \div b = c \quad \text{or} \quad \frac{a}{b} = c$$

variable $c$ is referred to as the **quotient** of $a$ and $b$, that is, $c$ is the result of dividing $a$ by $b$.

We define division in terms of multiplication:

$$a \div b = a \cdot \frac{1}{b} \quad \text{or} \quad \frac{a}{b} = a \cdot \frac{1}{b}$$

The two quantities $b$ and $1/b$ are **reciprocals** of each other. Reciprocals are two numbers whose product is 1:

$$b \times \frac{1}{b} = 1$$

If $a \div b = c$, then $b \times c = a$. One important consequence of this relationship is a prohibition against dividing by zero. *Division by zero is undefined* because there are no values for $a$ and $c$ that satisfy this equation:

$$\frac{a}{0} = c$$

If $a$, for example, is 25, then $c$ does not exist, because there is no value for $c$ that, when multiplied by zero, gives 25:

$$c \times 0 \neq 25$$

Of course, *zero divided by any number is zero* because any nonzero value for $b$ satisfies these equations:

$$\frac{0}{b} = 0 \quad \text{or} \quad b \times 0 = 0$$

Because we define division in terms of multiplication, the sign rules are the same. Table 1-2 ★ summarizes those rules.

★ **TABLE 1-2** The Four Sign Combinations in Division

| Rule |
|---|
| positive ÷ positive = positive |
| positive ÷ negative = negative |
| negative ÷ positive = negative |
| negative ÷ negative = positive |

**☑ CHECKPOINT 1-4**

Evaluate the following expressions.

(a) $\frac{18}{-3}$ (b) $2.4 \div 0.3$ (c) $\frac{-160}{-4}$ (d) $(-49) \div 7$

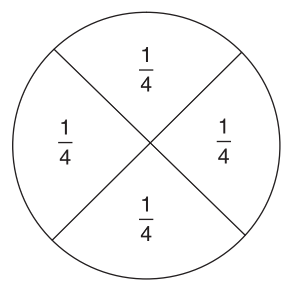
(e) $5\left(\frac{1}{10}\right)$ (f) $\frac{0.54}{-9}$ (g) $-35\left(\frac{1}{7}\right)$ (h) $25 \div (-75)$

(a) −6 (b) 8 (c) 40 (d) −7

(e) 0.5 (f) −0.06 (g) −5 (h) −0.33

## FRACTIONS

A fraction is nothing more than a representation of a division. The top number is the **numerator** and the bottom number is the **denominator**. The denominator specifies the number of equal parts into which we divide something, and the numerator specifies the number of those equal parts.




In the above diagram, for example, we divide the circle into four equal parts, and each part is one of the four. For each part, therefore, the denominator is 4 and the numerator is 1.

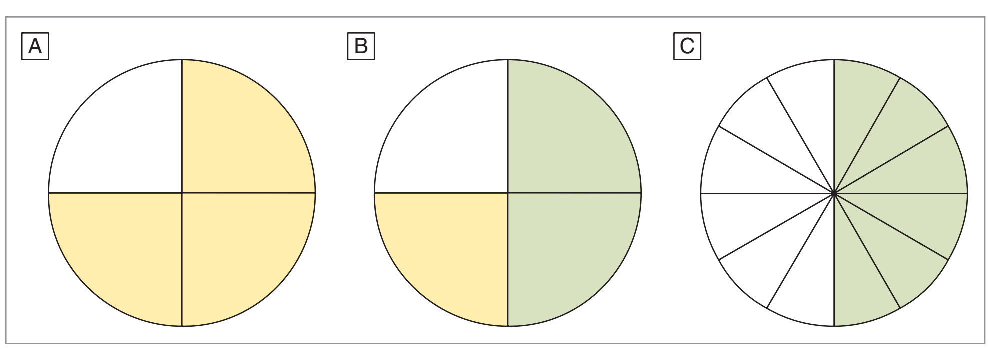
As a division, the fraction "$^1/_4$" tells us that (1) we divided one whole thing (a circle in this case) into four equal parts, and (2) we are considering one of those parts.

### Multiplying Fractions

*To multiply fractions, multiply the numerators and multiply the denominators.* For example,

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

What this equation tells us is that 2/3 of 3/4 is the same as 6/12. Figure 1-1 ■ depicts this relationship.




■ **FIGURE 1-1** A depiction of the equation $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$. Two-thirds of 3/4 is the same as 6/12. Panel *A:* Three-fourths of the circle is yellow. Panel *B:* This represents 2/3 of 3/4: 2/3 (in green) of the original 3/4 (in green and in yellow). Panel *C:* Six-twelfths (in green) of the whole circle, which is the same as the green area in panel *B*.

Multiplying a fraction by a whole number is straightforward; just treat the whole number as a fraction with "1" in the denominator. For example,

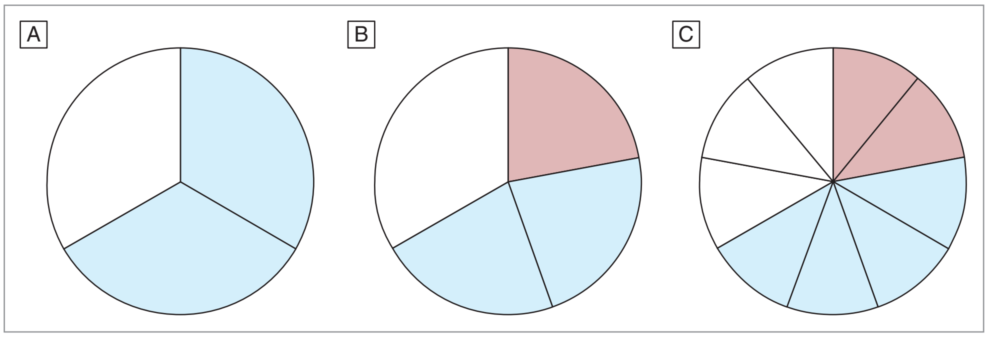
$$\frac{4}{5} \times 10 = \frac{4}{5} \times \frac{10}{1} = \frac{40}{5} = 8$$

## Dividing Fractions

*To divide a fraction, multiply it by the reciprocal of the other number:*

$$\frac{2}{3} \div 3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

What these equations tell us is that dividing 2/3 of an object into three equal parts gives 2/9 of that object. For example, consider a circle (Figure 1-2 ■).




■ **FIGURE 1-2** A depiction of the equations $\frac{2}{3} \div 3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$. Two-thirds divided by 3 is the same as 1/3 of 2/3, which equals 2/9. Panel *A:* Two-thirds of the circle is blue. Panel *B:* One-third (in purple) of the original 2/3 (in purple and in blue). Panel *C:* Two-ninths (in purple) of the whole circle, which is the same as the purple area in panel *B*.

### ☑ CHECKPOINT 1-5

1. Evaluate the following expressions.

   (a) $\frac{3}{5} \times \frac{4}{9}$ (b) $\frac{2}{7} \times \frac{1}{2}$ (c) $\frac{1}{4} \times \frac{2}{3}$ (d) $25 \times \frac{4}{5}$

2. Evaluate the following expressions.

   (a) $\frac{8}{9} \div 2$ (b) $\frac{1}{2} \div \frac{3}{5}$ (c) $6 \div \frac{2}{3}$ (d) $\frac{3}{7} \div \frac{4}{7}$

1. (a) $\frac{12}{45}$ (b) $\frac{2}{14}$ (c) $\frac{2}{12}$ (d) $25 \times \frac{4}{5} = \frac{100}{5} = 20$

2. (a) $\frac{8}{9} \div 2 = \frac{8}{9} \times \frac{1}{2} = \frac{8}{18}$ (b) $\frac{5}{6}$ (c) 9 (d) $\frac{21}{28}$

## Reducing Fractions

Generally, fractions should be **reduced** (or "simplified") so that the numerator and denominator are as small as possible, that is, until the only number evenly divisible into both of them is "1."

Sometimes the reduction is comparatively easy to see, as in the following example.

$$\frac{2}{4} \text{ reduces to } \frac{1}{2}$$

In the fraction $\frac{2}{4}$, the "2" divides evenly into the "4"; therefore, the "2" reduces to a "1" and the "4" reduces to a "2."

Here is another simple example:

$$\frac{5}{20} \text{ reduces to } \frac{1}{4}$$

The "5" divides evenly into the "20"; therefore, the "5" reduces to a "1" and the "20" reduces to a "4."

In more complex reductions, it helps to write out the factors. Three examples follow.

$$\frac{18}{32} = \frac{2}{2} \times \frac{9}{16} = 1 \times \frac{9}{16} = \frac{9}{16}$$

$$\frac{9}{15} = \frac{3}{3} \times \frac{3}{5} = 1 \times \frac{3}{5} = \frac{3}{5}$$

$$\frac{16}{64} = \frac{16}{16} \times \frac{1}{4} = 1 \times \frac{1}{4} = \frac{1}{4}$$

## Canceling

We can simplify operations on fractions by the shortcut known as **canceling**, which exploits simple reductions. For example, consider this problem and its long solution:

$$\frac{4}{5} \times \frac{15}{16} = \frac{4 \times 15}{5 \times 16} = \frac{15 \times 4}{5 \times 16} = \frac{15}{5} \times \frac{4}{16} = 3 \times \frac{1}{4} = \frac{3}{4}$$

Now consider the same problem simplified by canceling:

$$\frac{\cancel{4}^{1}}{{}_{1}\cancel{5}} \times \frac{\cancel{15}^{3}}{{}_{4}\cancel{16}} = \frac{1 \times 3}{1 \times 4} = \frac{3}{4}$$

The "4" in the numerator divides evenly into the "16" in the denominator; as a result, the "4" becomes a "1" and the "16" a "4." We say that the "4" cancels out. Likewise, the "5" in the denominator divides evenly into the "15" in the numerator; accordingly, the "5" becomes a "1" and the "15" a "3." We say that the "5" cancels out.

Here is another example:

$$\frac{\cancel{7}^{1}}{{}_{2}\cancel{16}} \times \frac{\cancel{8}^{1}}{{}_{3}\cancel{21}} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$$

### ☑ CHECKPOINT 1-6

Reduce the following fractions.

(a) $\frac{4}{6}$ (b) $\frac{16}{36}$ (c) $\frac{28}{56}$ (d) $\frac{9}{12}$ (e) $\frac{5}{20}$

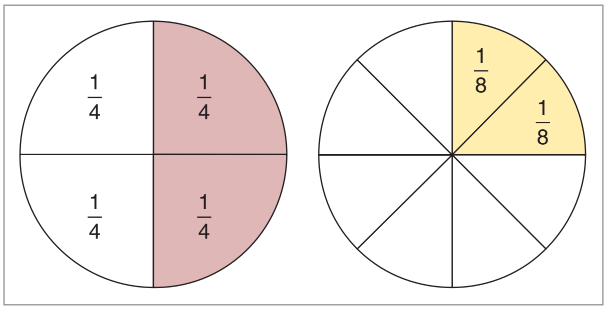
(a) $\frac{2}{3}$ (b) $\frac{4}{9}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ (e) $\frac{1}{4}$

## Adding and Subtracting Fractions

*To add (or subtract) fractions, add (or subtract) the numerators but not the denominators.* Furthermore, *the denominators must all be the same.*

Consider the simple addition of 1/4 and 1/4, which is highlighted in pink in the diagram below.

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$$




■ **FIGURE 1-3** Adding fractions entails adding the numerators but not the denominators. Clearly, the proportion of 2/4 (pink) is greater than the proportion of 2/8 (orange). Therefore, $\frac{1}{4} + \frac{1}{4} = \frac{2}{4} \neq \frac{2}{8}$.

It is logical to add only the numerators together because we clearly have two-fourths of the circle. As Figure 1-3 ■ shows, adding the denominators would be meaningless: it is impossible to arrive at two-eighths by adding together 1/4 and 1/4. Therefore, *adding or subtracting fractions requires a common denominator*.

If two denominators are different, we must equalize them before addition or subtraction. To accomplish this, we find the **least common denominator**, which is the single lowest number into which each denominator divides evenly. For example, in the problem

$$\frac{2}{3} + \frac{1}{4}$$

the least common denominator is "12." To prove this, we construct a chart of multiples:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Multiples of 3: | 3 | 6 | 9 | 12 | 15 | 18 | 21 |
| Multiples of 4: | 4 | 8 | 12 | 16 | 20 | 24 | 28 |

Therefore, the addition problem above becomes

$$\frac{g}{12} + \frac{h}{12}$$

The next step is to find the numerators $g$ and $h$ that correspond to the new denominator:

$$\frac{2}{3} = \frac{g}{12} \quad \text{and} \quad \frac{1}{4} = \frac{h}{12}$$

In the first equation (for numerator $g$), the original denominator of 3 was multiplied by 4 to give the least common denominator of 12. Therefore, we also multiply the numerator by 4:

$$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$$

In the second equation (for numerator $h$), the original denominator of 4 was multiplied by 3. Therefore, we also multiply the numerator by 3:

$$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$

Now we may perform the addition:

$$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$$